

A simple method for calculation of conditions behind shock waves

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1969 J. Phys. A: Gen. Phys. 2 415

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Comparison of the error terms in the lower bounds of the inequalities (14) and (15) for this particular case indicates that the latter lower bound is likely to be better than the former for small values of β .

These results not only include all the results derived earlier in I and II and also those derived by a number of other authors (see, for example, Robinson 1969a, b where further references will be found) as particular cases but go very much farther.

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3rd February 1969

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J. PHYS. A (GEN. PHYS.), 1969, SER. 2, VOL. 2. PRINTED IN GREAT BRITAIN

A simple method for calculation of conditions behind shock waves

Abstract. Conditions behind the incident and reflected shock wave in a shock tube are calculated from the initial conditions of pressure, density and temperature, together with the known speed of the incident shock wave. In this method the enthalpy of the gas is regarded as consisting of two parts: (i) the enthalpy evaluated for a gas consisting of molecules having no internal excitation and (ii) a correction term to be added to (i) to take account of internal excitation. An iterative solution to the flow equations for incident and reflected shock waves based on this correction term is obtained. Results for carbon monoxide are given in graphical form.

In shock-tube research the need arises for calculating conditions in shock-heated gases from known initial conditions and the measured shock speed. Although tables are available for some gases, interpolation using a table is not always accurate enough. The method presented here is very well suited for computer use.

In a frame of reference in which the incident shock wave is at rest the equations of state, continuity, momentum and energy are, respectively,

$$p = \rho RT \quad (1)$$

$$\rho_2 u_2 = \rho_1 u_1 = m \quad (2)$$

$$p_2 + \rho_2 u_2^2 = p_1 + \rho_1 u_1^2 = i \quad (3)$$

$$h_2 + \frac{1}{2} u_2^2 = h_1 + \frac{1}{2} u_1^2 = h_0 \quad (4)$$

where h is given by

$$h = (1 + \frac{1}{2}n)RT + \phi(T). \quad (5)$$

p , ρ and T have their usual meanings, R is the gas constant per gramme, u is the flow speed and h is the specific enthalpy. Suffixes 1 and 2 refer to conditions upstream and downstream of the shock wave, respectively. Conditions 1 are assumed known. Clearly, u_1 is the speed of the shock wave in laboratory coordinates. n is the number of degrees of freedom for rigid molecules ($n = 3, 5$ or 6 according as to whether the gas particles are monatomic, linear molecules or non-linear molecules).

Equation (5) shows the enthalpy as consisting of two parts: (i) $(1 + \frac{1}{2}n)RT$, which is the enthalpy appropriate to rigid molecules, and (ii) the term $\phi(T)$, which represents the

contribution to the enthalpy due to internal excitation of the molecules. There is also a pressure-dependent part to the enthalpy but, under many shock-tube conditions, this is very small and is neglected here. However, this part of the correction could be included without changing the principles of the method. m , i and h_0 are clearly known constants of the flow.

From these equations the following two equations may be derived:

$$T_2 = \left(\frac{i}{m} - u_2\right) \frac{u_2}{R} \quad (6)$$

$$u_2 = \frac{(1 + \frac{1}{2}n)i/m - \left[\left\{(1 + \frac{1}{2}n)i/m\right\}^2 - 2(n+1)\{h_0 - \phi(T_2)\}\right]^{1/2}}{n+1}. \quad (7)$$

As a first approximation, we put $\phi(T_2) = 0$ in (7) (i.e. we assume no internal excitation of the molecules) and the first approximation to u_2 is calculated. This is then used in (6) to derive T_2 . With a knowledge of T_2 , $\phi(T_2)$ can be determined from enthalpy tables. This new $\phi(T_2)$ is substituted in (7) and the cycle repeated until the result has converged to sufficient accuracy. The remaining conditions 2 are then simple to calculate.

The equations of continuity, momentum and energy for a frame of reference in which the reflected shock wave is at rest are

$$\rho_3 u_3 = \rho_2 (u_3 + U) \quad (8)$$

where

$$U = u_1 - u_2$$

$$\dot{p}_3 + \rho_3 u_3^2 = \dot{p}_2 + \rho_2 (u_3 + U)^2 \quad (9)$$

$$h_3 + \frac{1}{2}u_3^2 = h_2 + \frac{1}{2}(u_3 + U)^2 \quad (10)$$

where suffixes 2 and 3 refer to conditions upstream and downstream of the shock, respectively. u_3 is the reflected shock speed in laboratory coordinates. Conditions 2 are assumed known from the incident shock calculations.

Equations (8), (9) and (10) are rearranged to give

$$(\rho_3 - \rho_2)u_3 = \rho_2 U = M \quad (8a)$$

$$\dot{p}_3 + (\rho_3 - \rho_2)u_3^2 - 2\rho_2 u_3 U = \dot{p}_2 + \rho_2 U^2 = I \quad (9a)$$

$$h_3 - u_3 U = h_2 + \frac{1}{2}U^2 = H_0. \quad (10a)$$

M , I and H_0 are known quantities obtained from calculations on the incident shock wave. From these equations, together with the equation of state (1) and the enthalpy equation (5), the following two equations may be derived:

$$T_3 = \left(\frac{I}{M} + u_3\right) \frac{U u_3}{R(u_3 + U)} \quad (11)$$

$$u_3 = -\frac{1}{n} \left\{ \left(1 + \frac{1}{2}n\right) \frac{I}{M} - U - \frac{H_0 - \phi(T_3)}{U} \right\} + \frac{1}{n} \left[\left\{ \left(1 + \frac{1}{2}n\right) \frac{I}{M} - U - \frac{H_0 - \phi(T_3)}{U} \right\}^2 + 2n\{H_0 - \phi(T_3)\} \right]^{1/2}. \quad (12)$$

The calculation proceeds as for the incident shock wave: a first approximation to u_3 is found by putting $\phi(T_3) = 0$ and this is used as the starting point for the iteration.

The enthalpy tables used in this work are those of Hilsenrath *et al.* (1960). It was found convenient to represent the enthalpy data by a best-fit curve using Chebyshev polynomials. A best-fit function involving Chebyshev polynomials up to and including degree 10 fitted the enthalpy data in the temperature range 250 °K to 5000 °K to better than three significant figures. No account has been taken of dissociation or ionization, but these effects can be included with some modification to the basic method provided that there are sufficient thermodynamic data for the gas concerned.

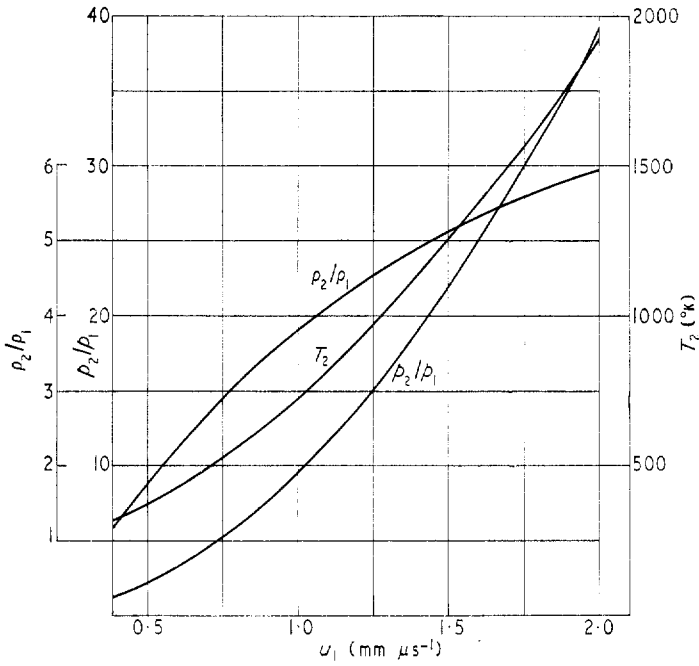


Figure 1. Conditions behind incident shock wave in carbon monoxide ($T_1 = 293$ $^{\circ}K$).

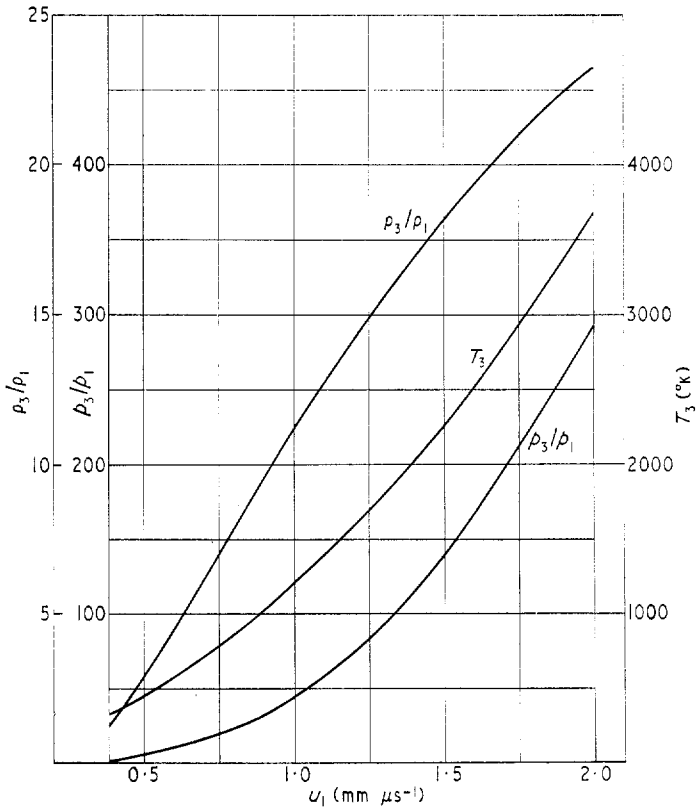


Figure 2. Conditions behind reflected shock wave in carbon monoxide ($T_1 = 293$ $^{\circ}K$).

Figures 1 and 2 show the conditions attained behind incident and reflected shock waves in carbon monoxide over a range of shock speeds.

Note added in proof. Successive iterations to the solutions of equations (6), (7) and (11), (12) oscillate about the true solution: convergence is greatly assisted if each new solution is taken as the arithmetic mean of the previous two solutions.

The work described in this letter has been carried out at the National Physical Laboratory. The author is grateful to Dr. D. Schofield for the curve-fitting programme used to determine the best-fit curve to the enthalpy data.

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24th February 1969

HILSEN RATH, J., *et al.*, 1960, *Tables of Thermodynamic and Transport Properties of Air, Argon, Carbon Dioxide, Carbon Monoxide, Hydrogen, Nitrogen, Oxygen and Steam* (Oxford: Pergamon Press).